

# Reconstruction of Domain Wall Universe and Localization of Gravity

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We construct a four-dimensional domain wall universe by using the Brans-Dicke type gravity with two scalar field. We give a formulation where for arbitrarily given warp factor and scale factor, we construct an action which reproduces both of the warp and scale factors as a solution of the Einstein equation and the field equations given by the action. This formulation could be called as reconstruction. We show that the model does not contain ghost with negative kinetic term and there occur the localization of gravity as in the Randall-Sundrum model. It should be noted that in the equation of the graviton, there appear extra terms related with the extra dimension, which might affect the tensor mode in the fluctuations in the universe.

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## I. INTRODUCTION

Many scenarios that our universe could be a brane in the higher dimensional space-time have been proposed [1–6]. Especially trace anomaly was used for the inflationary brane world models [7–9]. The brane may be given by a limit where the thickness of the domain wall vanishes. In fact the domain world scenario, where we live on on the domain wall with finite thickness (thick brane), was proposed in [10] before the proposition of the brane world scenario and bent domain wall [11] as well as dynamical domain wall [12] were also investigated. After that there were many activities in the domain wall or thick brane universe scenario [13–18]. Recently in [19], it has been proposed a domain wall model with two scalar fields and it has been shown that we can construct a model which generates space-time, where the scale factor of the domain wall universe, which could be the general FRW universe, and the warp factor are arbitrarily given. Such a formulation can be a generalization of the reconstruction of the domain wall [20]. A formulation, where only the warp factor of the domain wall can be arbitrary, was proposed in the seminal paper [13]. In the model [19], the scalar field equations can be identified with the Bianchi identities:  $\nabla^\mu (R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}) = 0$  and therefore the equations can be satisfied automatically.

A critical problem in the formulation of [19] is that there appears a ghost scalar field, whose kinetic term has non-canonical signature, in general although a model without ghost was proposed. In this paper, we show that we can construct a model which is ghost free and reproduces given scale and warp factors, by using the Brans-Dicke type model where a scalar field couples with the scalar curvature directly in the action. Furthermore we show that the graviton is localized on the domain wall and there appears massless graviton propagating on the domain wall in the model proposed in [19]. The massless graviton on the four dimensional brane world corresponds to the zero mode of the graviton in the five dimensional space-time. This tells that the graviton is localized on the domain wall even in the Brans-Dicke type model proposed in this paper. In the equation of the graviton, however, there appear extra terms related with the extra dimension, which may affect the tensor mode of the fluctuation in the universe.

In the next section, we review on the formulation in [19]. In section III, we construct the Brans-Dicke type model without ghost. In section IV, we observe that the graviton is localized on the domain wall but in the equation of the graviton, there appear extra terms. The last section V is devoted with the discussions.

## II. DOMAIN WALL MODEL WITH TWO SCALAR FIELDS

In this section, we review the formulation of the domain wall model with two scalar fields based on [19]. The formulation is a generalization of the formulation in [21], where it was shown how we can construct models which admit the exact solutions describing the domain wall with given warp factor, and it was used a procedure for a scale factor in [22].

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### A. Reconstruction of general FRW domain wall universe

We now investigate general domain wall, which can be regarded as a general FRW universe, and the metric of the space-time in five dimensions is given by

$$ds^2 = dw^2 + f(w, t) \left\{ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right\} - \frac{e(w, t)^2}{f(w, t)} dt^2$$

$$\equiv e^{\ln f(w, t)} \gamma_{mn}(x) dx^m dx^n + h_{\alpha\beta}(y) dy^\alpha dy^\beta. \quad (2.1)$$

Here  $m, n = 1, 2, 3$ ,  $\alpha, \beta = 0, 5$ , and  $y^0 = t$ ,  $y^5 = w$ . By choosing

$$f(w, t) = L^2 e^{u(w, t)} a(t)^2, \quad e(w, t) = L^2 e^{u(w, t)} a(t), \quad (2.2)$$

we find that the general FRW universe, whose metric is

$$ds_{\text{FRW}}^2 = -dt^2 + a(t)^2 \left\{ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right\}, \quad (2.3)$$

is embedded by the arbitrary warp factor  $L^2 e^{u(w, t)}$  in the five dimensional space-time.

The following action with two scalar fields  $\phi$  and  $\chi$  was proposed in [19]:

$$S_{\phi\chi} = \int d^5 x \sqrt{-g} \left\{ \frac{R}{2\kappa^2} - \frac{1}{2} A(\phi, \chi) \partial_\mu \phi \partial^\mu \phi - B(\phi, \chi) \partial_\mu \phi \partial^\mu \chi - \frac{1}{2} C(\phi, \chi) \partial_\mu \chi \partial^\mu \chi - V(\phi, \chi) \right\}. \quad (2.4)$$

We can construct a model to realize the arbitrary metric written in the form of (2.1).

For the model (2.4), the energy-momentum tensor could be given by

$$T_{\mu\nu}^{\phi\chi} = g_{\mu\nu} \left\{ -\frac{1}{2} A(\phi, \chi) \partial_\rho \phi \partial^\rho \phi - B(\phi, \chi) \partial_\rho \phi \partial^\rho \chi - \frac{1}{2} C(\phi, \chi) \partial_\rho \chi \partial^\rho \chi - V(\phi, \chi) \right\}$$

$$+ A(\phi, \chi) \partial_\mu \phi \partial_\nu \phi + B(\phi, \chi) (\partial_\mu \phi \partial_\nu \chi + \partial_\nu \phi \partial_\mu \chi) + C(\phi, \chi) \partial_\mu \chi \partial_\nu \chi. \quad (2.5)$$

On the other hand, the field equations read

$$0 = \frac{1}{2} A_\phi \partial_\mu \phi \partial^\mu \phi + A \nabla^\mu \partial_\mu \phi + A_\chi \partial_\mu \phi \partial^\mu \chi + \left( B_\chi - \frac{1}{2} C_\phi \right) \partial_\mu \chi \partial^\mu \chi + B \nabla^\mu \partial_\mu \chi - V_\phi, \quad (2.6)$$

$$0 = \left( -\frac{1}{2} A_\chi + B_\phi \right) \partial_\mu \phi \partial^\mu \phi + B \nabla^\mu \partial_\mu \phi + \frac{1}{2} C_\chi \partial_\mu \chi \partial^\mu \chi + C \nabla^\mu \partial_\mu \chi + C_\phi \partial_\mu \phi \partial^\mu \chi - V_\chi. \quad (2.7)$$

Here  $A_\phi = \partial A(\phi, \chi) / \partial \phi$ , etc. We now choose  $\phi = t$  and  $\chi = w$ . Then we find

$$T_0^0 = -\frac{f}{2e^2} A - \frac{1}{2} C - V, \quad T_i^j = \delta_i^j \left( \frac{f}{2e^2} A - \frac{1}{2} C - V \right), \quad T_5^5 = \frac{f}{2e^2} A + \frac{1}{2} C - V, \quad T_0^5 = B, \quad (2.8)$$

and

$$0 = -\frac{f}{2e^2} A_\phi + \frac{f}{e^2} \left( \frac{\dot{e}}{e} - \frac{2\dot{f}}{f} \right) A + B_\chi + B \left( \frac{e'}{e} + \frac{f'}{f} \right) - \frac{1}{2} C_\phi - V_\phi, \quad (2.9)$$

$$0 = \frac{f}{2e^2} A_\chi - \frac{f}{e^2} B_\phi + \frac{f}{e^2} \left( \frac{\dot{e}}{e} - \frac{2\dot{f}}{f} \right) B + \frac{1}{2} C_\chi + C \left( \frac{e'}{e} + \frac{f'}{f} \right) - V_\chi. \quad (2.10)$$

We may solve Eqs. (2.8) with respect to  $A$ ,  $B$ ,  $C$ ,  $V$  and obtain

$$\begin{aligned}
A &= \frac{e^2}{\kappa^2 f} (G_1^1 - G_0^0) = \frac{e^2}{\kappa^2 f} (G_2^2 - G_0^0) = \frac{e^2}{\kappa^2 f} (G_3^3 - G_0^0) \\
&= \frac{1}{\kappa^2} \left( -\frac{e^2 f''}{f^2} + \frac{ee''}{f} + \frac{2ke^2}{f^2} - \frac{\ddot{f}}{f} + \frac{\dot{f}^2}{2f^2} + \frac{\dot{e}\dot{f}}{ef} \right), \\
B &= \frac{1}{\kappa^2} G_0^5 = \frac{1}{\kappa^2} \left( -\frac{3e'\dot{f}}{2e^3} + \frac{3\dot{f}'}{2e^2} \right), \\
C &= \frac{1}{\kappa^2} (G_5^5 - G_1^1) = \frac{1}{\kappa^2} (G_5^5 - G_2^2) = \frac{1}{\kappa^2} (G_5^5 - G_3^3) \\
&= \frac{1}{\kappa^2} \left( -\frac{f''}{2f} - \frac{e''}{e} - \frac{2k}{f} - \frac{\ddot{f}}{2e^2} - \frac{\dot{f}^2}{2e^2 f} + \frac{\dot{e}\dot{f}}{2e^3} + \frac{3f'e'}{2fe} \right), \\
V &= \frac{1}{\kappa^2} (G_0^0 + G_5^5) \\
&= \frac{1}{\kappa^2} \left( -\frac{3f''}{4f} + \frac{3k}{f} + \frac{3\dot{f}^2}{4e^2 f} - \frac{3f'e'}{4fe} + \frac{3\ddot{f}}{4e^2} - \frac{3\dot{e}\dot{f}}{4e^3} \right). \tag{2.11}
\end{aligned}$$

Here  $G_{\mu\nu}$  is the Einstein tensor. Therefore the equations (2.9) and (2.10) are nothing but the Bianchi identities:

$$-\frac{f}{2e^2} A_\phi + \frac{f}{e^2} \left( \frac{\dot{e}}{e} - \frac{2\dot{f}}{f} \right) A + B_\chi + B \left( \frac{e'}{e} + \frac{f'}{f} \right) - \frac{1}{2} C_\phi - V_\phi = -\frac{e^2}{2f} \nabla^\mu G_\mu^0, \tag{2.12}$$

$$\frac{f}{2e^2} A_\chi - \frac{f}{e^2} B_\phi + \frac{f}{e^2} \left( \frac{\dot{e}}{e} - \frac{2\dot{f}}{f} \right) B + \frac{1}{2} C_\chi + C \left( \frac{e'}{e} + \frac{f'}{f} \right) - V_\chi = \nabla^\mu G_\mu^5. \tag{2.13}$$

Therefore the equations (2.9) and (2.10) are automatically satisfied by choosing  $A$ ,  $B$ ,  $C$ , and  $V$  by (2.11).

### B. An example of reconstructed model

In the action (2.4), when one of the eigenvalue of the matrix  $\begin{pmatrix} A & B \\ B & C \end{pmatrix}$  is negative, there appears a ghost, which conflicts with the quantum mechanics (see [23], for example). In [19], some examples, where no ghost field appears, were given in case  $k = 0$ , just for simplicity.

As an example, we may assume  $a(t) \propto t^{h_0}$  with a constant  $h_0$ . Then, the equations in (2.11) give,

$$\begin{aligned}
\kappa^2 A &= -\frac{\ddot{U}}{U} + \frac{3\dot{U}^2}{2U^2} + \frac{h_0\dot{U}}{tU} + \frac{2h_0}{t^2}, \quad L^2 \kappa^2 B = -\frac{3U'\dot{U}}{2U^3} + \frac{3\dot{U}'}{2U^2}, \\
\kappa^2 C &= -\frac{3U''}{2U} + \frac{3U'^2}{2U^2} + \frac{1}{L^2} \left( -\frac{\ddot{U}}{2U^2} - \frac{5h_0\dot{U}}{2tU^2} - \frac{3h_0^2 - h_0}{t^2 U} \right). \tag{2.14}
\end{aligned}$$

Here  $U(w, t) \equiv e^{u(w, t)}$ . By further assuming  $U(w, t) = W(w)T(t)$ , Eq. (2.14) can be rewritten as

$$\begin{aligned}
\kappa^2 A &= -\frac{\ddot{T}}{T} + \frac{3\dot{T}^2}{2T^2} + \frac{h_0\dot{T}}{tT} + \frac{2h_0}{t^2}, \quad L^2 \kappa^2 B = 0, \\
\kappa^2 C &= -\frac{3W''}{2W} + \frac{3W'^2}{2W^2} + \frac{1}{L^2 W T^2} \left( -\frac{\ddot{T}}{2} - \frac{5h_0\dot{T}}{2t} - \frac{(3h_0^2 - h_0)T}{t^2} \right). \tag{2.15}
\end{aligned}$$

We now also assume  $T \propto t^\beta$  and obtain

$$\begin{aligned}
-\frac{\ddot{T}}{2} - \frac{5h_0\dot{T}}{2t} - \frac{(3h_0^2 - h_0)T}{t^2} &\propto -\frac{1}{2} (\beta^2 - (1 - 5h_0)\beta + 6h_0^2 - 2h_0) \\
&= -\frac{1}{2} \{\beta + (3h_0 - 1)\} \{\beta + 2h_0\}. \tag{2.16}
\end{aligned}$$

This tells that  $T$  is given by

$$T(t) = T_1 t^{1-3h_0} + T_2 t^{-2h_0}, \quad (2.17)$$

and we obtain

$$A = \frac{1}{\kappa^2} \left\{ \frac{3}{2} \left( \frac{\dot{T}(t)}{T(t)} + \frac{2h_0}{t} \right)^2 \right\} > 0, \quad \kappa^2 C = -\frac{3W''}{2W} + \frac{3W'^2}{2W^2} = -\frac{3}{2} (\ln W)'' . \quad (2.18)$$

Then if  $\ln W$  is convex,  $C$  becomes positive. If we choose

$$W(w) = e^{-\sqrt{1+\frac{w^2}{w_0^2}}}, \quad (2.19)$$

with a constant  $w_0$ , we find

$$C = \frac{3}{2\kappa^2} \frac{\frac{1}{w_0^2}}{\left(1 + \frac{w^2}{w_0^2}\right)^{\frac{3}{2}}} > 0. \quad (2.20)$$

Because both of  $A$  and  $C$  are positive and  $B$  vanishes, any ghost does not appear in this model. Since  $a(t) \propto t^{h_0}$ , the domain universe corresponds to the universe filled with the perfect fluid whose equation of state parameter  $w$  is given by

$$w = -1 + \frac{2}{3h_0}. \quad (2.21)$$

Since we now have  $f(w, t) = L^2 T(t) e^{-\sqrt{1+\frac{w^2}{w_0^2}} a_0^2 t^{2h_0}}$ ,  $e(w, t) = L^2 T(t) e^{-\sqrt{1+\frac{w^2}{w_0^2}} a_0 t^{h_0}}$  in (2.2). Then by using the last equation in (2.11), we find the explicit form of the potential  $V$ :

$$\begin{aligned} V &= -\frac{3}{4\kappa^2} \left[ \frac{W''}{W} + \frac{W'^2}{W^2} - \frac{1}{L^2 W T} \left( \frac{\ddot{T}}{T} + \frac{5\dot{a}\dot{T}}{aT} + \frac{2\ddot{a}}{a} + \frac{4\dot{a}^2}{a^2} \right) \right] \\ &= -\frac{3}{4\kappa^2} \frac{1}{w_0^2} \left[ 2 - \frac{2}{1 + \frac{\chi^2}{w_0^2}} - \left( 1 + \frac{\chi^2}{w_0^2} \right)^{-\frac{3}{2}} \right]. \end{aligned} \quad (2.22)$$

By replacing  $w$  by  $\chi$ , we have obtained the explicit form of the potential  $V(\chi)$  in terms of the scalar field  $\chi$ . We should also note that we obtain a brane in the limit of  $w_0 \rightarrow 0$ .

### III. BRANS-DICKE TYPE MODEL

As mentioned at the beginning of Subsection IIB, the action (2.4) includes ghost field in general. In this section, by using the Brans-Dicke type model, we consider a formulation where arbitrary FRW universe is realized without ghost. For the purpose, we use the example in Subsection IIB, which does not include any ghost. When we consider the model, whose metric is given by scaling the metric in the model without ghost, the model does not include ghost, either.

In this section we work by using the conformal time  $\tau$ , which is related with the cosmological time  $t$  in (2.1) or (2.3) by

$$d\tau = \frac{dt}{a(t)}, \quad (3.1)$$

and the FRW metric (2.3) can be rewritten as

$$ds_{\text{FRW}}^2 = a(t(\tau))^2 \left[ -d\tau^2 + \{dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2\} \right], \quad (3.2)$$

In this section, we only consider  $k = 0$  case, for simplicity, again. In case of Subsection II B,  $a(t) = a_0 t^{h_0}$  with constants  $a_0$  and  $h_0$  and we find the explicit relation between  $\tau$  and  $t$ , as follows

$$\tau = \frac{t^{1-h_0}}{(1-h_0)a_0}. \quad (3.3)$$

Then if we define a new scalar field  $\varphi$  by

$$\varphi = \varphi(t) \equiv \frac{\phi^{1-h_0}}{(1-h_0)a_0}, \quad (3.4)$$

we find

$$\varphi = \tau, \quad a(t) = \tilde{a}(\tau) = a_0 ((1-h_0)a_0\tau)^{\frac{h_0}{1-h_0}}. \quad (3.5)$$

Then arbitrary warp factor  $a(t(\tau)) = A(\tau)$  can be realized by multiplying a function with the metric  $g_{\mu\nu}$  by  $\frac{A(\tau)^2}{\tilde{a}(t(\tau))^2} = e^{2\Theta(t)}$  or

$$g_{\mu\nu} \rightarrow e^{-2\Theta(\phi)} g_{\mu\nu}. \quad (3.6)$$

Then since

$$R \rightarrow e^{2\Theta(\phi)} (R + 8\Box\Theta(\phi) - 12\partial_\mu\Theta(\phi)\partial^\mu\Theta(\phi)), \quad (3.7)$$

in five dimensions, the action (2.4) can be rewritten as

$$\begin{aligned} S_{\phi\chi} &= \int d^5x \sqrt{-g} \left\{ \frac{e^{-3\Theta(\phi)} R}{2\kappa^2} - \frac{1}{2} e^{-3\Theta(\phi)} \left( A(\phi, \chi) - \frac{12}{\kappa^2} \Theta'(\phi)^2 \right) \partial_\mu \phi \partial^\mu \phi - e^{-3\Theta(\phi)} B(\phi, \chi) \partial_\mu \phi \partial^\mu \chi \right. \\ &\quad \left. - \frac{1}{2} e^{-3\Theta(\phi)} C(\phi, \chi) \partial_\mu \chi \partial^\mu \chi - e^{-5\Theta(\phi)} V(\phi, \chi) \right\} \\ &= \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} \left\{ \left( \frac{a_0 \phi^{h_0}}{A(\phi)} \right)^3 R - 3 \left( \frac{a_0 \phi^{h_0}}{A(\phi)} \right)^3 \left[ \frac{1}{2} \left( \frac{\dot{T}(\phi)}{T(\phi)} + \frac{2h_0}{\phi} \right)^2 - 4 \left( \frac{A'(\phi)}{A(\phi)} - \frac{h_0}{\phi} \right)^2 \right] \partial_\mu \phi \partial^\mu \phi \right. \\ &\quad \left. - \frac{3}{2w_0^2} \left( 1 + \frac{\chi^2}{w_0^2} \right)^{-\frac{3}{2}} \left( \frac{a_0 \phi^{h_0}}{A(\phi)} \right)^3 \partial_\mu \chi \partial^\mu \chi + \frac{3}{2w_0^2} \left( \frac{a_0 \phi^{h_0}}{A(\phi)} \right)^5 \left[ 2 - \frac{2}{1 + \frac{\chi^2}{w_0^2}} - \left( 1 + \frac{\chi^2}{w_0^2} \right)^{-\frac{3}{2}} \right] \right\}. \quad (3.8) \end{aligned}$$

This action can be regarded as the action in the Jordan frame action. Let assume that the particles in the standard model of the elementary particle physics couple with the metric in the Jordan frame and do not couple with the scalar field  $\phi$  nor  $\chi$ . Then if we start with the action (3.8), we can realized any history of the expansion of the universe given by  $a(t(\tau)) = A(\tau)$ .

#### IV. LOCALIZATION OF GRAVITON

In the second Randall-Sundrum model [3], the massless graviton is localized on the brane. The massless graviton corresponds to the zero mode of the graviton in five dimensions. In [19], it has been shown that the graviton can be localized on the domain wall, which corresponds to the flat, de Sitter, or anti-de Sitter space-time. In this section, we show that the localization of the graviton could occur in the model (2.4), which tells that the localization occurs even in the Brans-Dicke type model (3.8), since we do not change the warp factor in the scale transformation (3.6). An interesting point is that there appears a correction proportional to the derivative of the warp factor with respect the time in the equation of the graviton. This correction may affect the tensor mode of the fluctuation in the universe. Before going to the problem of the localization, we need to find the equation of the graviton in the FRW universe in four dimensions. In order to find the equation, we consider the model where a scalar field coupled with gravity. After that we compared the equation for the zero mode of the graviton in five dimensions with the equation of the graviton in four dimensions.

### A. Equation of graviton in four dimensional FRW universe

We now consider the following perturbation

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + h_{\mu\nu}. \quad (4.1)$$

Then we obtain

$$\delta R_{\mu\nu} = \frac{1}{2} [\nabla_\mu \nabla^\rho h_{\nu\rho} + \nabla_\nu \nabla^\rho h_{\mu\rho} - \nabla^2 h_{\mu\nu} - \nabla_\mu \nabla_\nu (g^{\rho\lambda} h_{\rho\lambda}) - 2R^\lambda{}_\nu{}^\rho{}_\mu h_{\lambda\rho} + R^\rho{}_\mu h_{\mu\nu} + R^\rho{}_\nu h_{\rho\mu}], \quad (4.2)$$

$$\delta R = -h_{\mu\nu} R^{\mu\nu} + \nabla^\mu \nabla^\nu h_{\mu\nu} - \nabla^2 (g^{\mu\nu} h_{\mu\nu}). \quad (4.3)$$

By imposing the gauge condition

$$\nabla^\mu h_{\mu\nu} = g^{\mu\nu} h_{\mu\nu} = 0, \quad (4.4)$$

the Einstein equation  $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa^2 T_{\mu\nu}$  has the following form:

$$\frac{1}{2} [-\nabla^2 h_{\mu\nu} - 2R^\lambda{}_\nu{}^\rho{}_\mu h_{\lambda\rho} + R^\rho{}_\mu h_{\mu\nu} + R^\rho{}_\nu h_{\rho\mu} - h_{\mu\nu}R + g_{\mu\nu}R^{\rho\lambda}h_{\rho\lambda}] = \kappa^2 \delta T_{\mu\nu}. \quad (4.5)$$

We may consider scalar field theory in [24], whose action is given by

$$S_\phi = \int d^4x \sqrt{-g} \mathcal{L}_\phi, \quad \mathcal{L}_\phi = -\frac{1}{2}\omega(\phi)\partial_\mu\phi\partial^\mu\phi - V(\phi). \quad (4.6)$$

Then we obtain

$$T_{\mu\nu} = -\omega(\phi)\partial_\mu\phi\partial_\nu\phi + g_{\mu\nu}\mathcal{L}_\phi, \quad (4.7)$$

and therefore

$$\delta T_{\mu\nu} = h_{\mu\nu}\mathcal{L}_\phi + \frac{1}{2}g_{\mu\nu}\omega(\phi)\partial^\rho\phi\partial_\rho\phi. \quad (4.8)$$

Because we are now interested in the graviton, we may assume  $h_{\mu\nu} = 0$  except the components with  $\mu, \nu = 1, 2, 3$ . In the FRW universe (2.3) with  $k = 0$ , we may assume  $\phi = t$  in (4.8), Then by using the FRW equation

$$\frac{3}{\kappa^2} \frac{\dot{a}^2}{a^2} = \frac{\omega}{2} + V, \quad \frac{1}{\kappa^2} \left( 2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) = \frac{\omega}{2} - V, \quad (4.9)$$

we find

$$\omega = \frac{1}{\kappa^2} \left( 2\frac{\ddot{a}}{a} + 4\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right), \quad V = -\frac{1}{\kappa^2} \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right). \quad (4.10)$$

By using (4.5), (4.8), (4.10), and  $\phi = t$ , we find the equation of graviton:

$$0 = \left( 2\frac{\ddot{a}}{a} + \frac{\dot{a}}{a}\partial_t - \partial_t^2 + \frac{\Delta}{a^2} \right) h_{ij}. \quad (4.11)$$

### B. Localization of the graviton on the domain wall

In this subsection, we show that the graviton can be localized on the domain wall or there exists a zero mode solution in the equation for the graviton in five dimensions. Explicit formulas for the connection and curvatures are given in Appendix.

By using (2.8) and (2.9) or

$$\begin{aligned} \kappa^2 A &= -\ddot{u} + \frac{1}{2}\dot{u}^2 - 2\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + \frac{\dot{a}\dot{u}}{a} + 2\frac{k}{a^2}, \\ \kappa^2 B &= -\frac{3}{2}\dot{u}', \\ \kappa^2 C &= -\frac{3}{2}u'' - \frac{1}{2}L^{-2}e^{-u} \left( \ddot{u} + \dot{u}^2 + 2\frac{\ddot{a}}{a} + 5\frac{\dot{a}\dot{u}}{a} + 4\frac{\dot{a}^2}{a^2} + 4\frac{k}{a^2} \right), \\ \kappa^2 V &= -\frac{3}{4} \left[ u'' + 2u'^2 - L^{-2}e^{-u} \left( \ddot{u} + \dot{u}^2 + 2\frac{\ddot{a}}{a} + 4\frac{\dot{a}^2}{a^2} + 5\frac{\dot{a}\dot{u}}{a} + 4\frac{k}{a^2} \right) \right], \end{aligned} \quad (4.12)$$

we find the equation for graviton in five dimensions:

$$0 = \left[ \partial_w^2 - u'' - u'^2 + L^{-2} e^{-u} \left( \ddot{u} + \frac{\dot{a}\dot{u}}{a} + \dot{u}\partial_t + 2\frac{\ddot{a}}{a} + \frac{\dot{a}}{a}\partial_t - \partial_t^2 + \frac{\Delta}{a^2} \right) \right] h_{ij}. \quad (4.13)$$

By assuming  $h_{ij}(w, x) = e^{u(w, t)} \hat{h}_{ij}(x)$ , we find the following equation:

$$0 = \left( 2\frac{\dot{a}\dot{u}}{a} - \dot{u}\partial_t + 2\frac{\ddot{a}}{a} + \frac{\dot{a}}{a}\partial_t - \partial_t^2 + \frac{\Delta}{a^2} \right) \hat{h}_{ij}. \quad (4.14)$$

Then if  $u$  goes to minus infinity sufficiently rapidly for large  $|w|$ ,  $h_{ij}(w, x)$  is normalized in the direction of  $w$  and therefore there occurs the localization of graviton.

We should also note that if  $\dot{u}(2\frac{\dot{a}}{a} - \partial_t) \hat{h}_{ij} = 0$ , the above expression (4.14) coincides with the equation for the graviton in (4.11). Especially when the warp factor does not depend on time, that is,  $\dot{u} = 0$ , two expressions coincide with each other. Conversely if  $\dot{u} \neq 0$ , there could appear some corrections proportional to  $\dot{u}$  when we consider the tensor perturbation.

## V. DISCUSSIONS

By using the Brans-Dicke type gravity with two scalar field, we constructed a four-dimensional domain wall universe. In the formulation, when the warp factor and scale factor are arbitrarily given, we can construct an action which reproduces both of the warp and scale factors as an exact solution of the Einstein equation and the field equations given by the action. The obtained model does not contain ghost with negative kinetic term and there occur the localization of gravity as in the Randall-Sundrum model. In the equation of the graviton, there appear extra terms related with the extra dimension, which is proportional to the derivative of the warp factor with respect to the cosmological time  $t$ . This extra terms might affect the tensor mode in the fluctuations in the universe and might be found by the observations of the CMB and/or the structure formation of the universe.

The remaining problem is to investigate the stability of the domain wall solution, which requires the time-dependent perturbation from the solution. The existence of the massless graviton may tell that the model could be stable under the perturbation of the tensor mode in the metric. We need, of course, to include the perturbation of the scalar mode including the scalar fields  $\phi$  and  $\chi$ , in order to verify the stability, which could be a future work.

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#### 1. Explicit expressions of connections and curvatures in five dimensions

In this appendix, we give explicit expressions of connections and curvatures in five dimensional space-time, whose metric is given by

$$g_{AB} = \begin{pmatrix} -L^2 e^{u(w, t)} & & & & \\ & L^2 e^{u(w, t)} \frac{a(t)^2}{1 - kr^2} & & & \\ & & L^2 e^{u(w, t)} a(t)^2 r^2 & & \\ & & & L^2 e^{u(w, t)} a(t)^2 r^2 \sin^2 \theta & \\ & & & & 1 \end{pmatrix}. \quad (1)$$

Then the connections are given by

$$\begin{aligned} \Gamma_{tt}^t &= \frac{1}{2}\dot{u}, & \Gamma_{tt}^w &= \frac{1}{2}L^2 e^u u', & \Gamma_{rt}^r &= \Gamma_{\theta t}^\theta = \Gamma_{\phi t}^\phi = \frac{\dot{a}}{a} + \frac{1}{2}\dot{u}, & \Gamma_{tw}^t &= \Gamma_{rw}^r = \Gamma_{\theta w}^\theta = \Gamma_{\phi w}^\phi = \frac{1}{2}u', \\ \Gamma_{ij}^t &= L^{-2} e^{-u} \left( \frac{\dot{a}}{a} + \frac{1}{2}\dot{u} \right) g_{ij}, & \Gamma_{rr}^r &= \frac{kr}{1 - kr^2}, & \Gamma_{ij}^w &= -\frac{1}{2}u' g_{ij}, & \Gamma_{\theta r}^\theta &= \Gamma_{\phi r}^\phi = \frac{1}{r}, \\ \Gamma_{\theta\theta}^r &= -r(1 - kr^2), & \Gamma_{\phi\phi}^\phi &= \cot \theta, & \Gamma_{\phi\phi}^r &= -r(1 - kr^2) \sin^2 \theta, & \Gamma_{\phi\phi}^\theta &= -\cos \theta \sin \theta. \end{aligned} \quad (2)$$

The Ricci curvatures have the following forms:

$$\begin{aligned}
R_{tt} &= \left[ -\frac{1}{2}u'' - u'^2 + \frac{3}{2}L^{-2}e^{-u} \left( \ddot{u} + \frac{\dot{a}\dot{u}}{a} + 2\frac{\ddot{a}}{a} \right) \right] g_{tt}, \\
R_{ij} &= \left[ -\frac{1}{2}u'' - u'^2 + \frac{1}{2}L^{-2}e^{-u} \left( \ddot{u} + 5\frac{\dot{a}\dot{u}}{a} + 2\frac{\ddot{a}}{a} + 4\frac{\dot{a}^2}{a^2} + \dot{u}^2 + 4\frac{k}{a^2} \right) \right] g_{ij}, \\
R_{ww} &= -2u'' - u'^2 \\
R_{tw} &= -\frac{3}{2}\dot{u}'.
\end{aligned} \tag{3}$$

The scalar curvatures is

$$R = -4u'' - 5u'^2 + 3L^{-2}e^{-u} \left( \ddot{u} + \frac{1}{2}\dot{u}^2 + 3\frac{\dot{a}\dot{u}}{a} + 2\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + 2\frac{k}{a^2} \right). \tag{4}$$

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